Copula-based Collaborative Multi-Structure Damage Diagnosis and Prognosis for Fleet Maintenance Digital Twins

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I. Introduction

In aeronautical and mechanical structures, cyclic loading-induced fatigue leads to the emergence and growth of cracks in critical components, leading to structural failure and compromising structural integrity [1]. Building upon individual aircraft tracking, the airframe digital twin [2] facilitates structural damage diagnosis and prognosis through the creation of a multi-physical, multi-scale, and probabilistic virtual model of the system to support proactive fleet maintenance decisions [3, 4].

The particle filter (PF) has been widely used in airframe digital twin as it is capable of modeling non-Gaussian nonlinear processes that contain epistemic and aleatory uncertainties [5, 6]. However, current PF-based digital twin approaches primarily focus on individual diagnosis and prognosis, with little attention paid to the fleet level. In many cases, the damage state is correlated with different structures. It is desirable to develop an approach to efficiently consider the correlation between structures within the fleet and improve the holistic fleet diagnosis and prognosis.

In this paper, a novel copula-based approach is presented to address this challenge. The main contribution of this study is utilizing the copulas to model the dependence between crack length distributions to obtain an approximate joint probability distribution for collaborative updating. The resulting scheme is then integrated as a copula-based updating step into the particle filter, allowing for the updating of all structures in a fleet based on the observation of one individual structure.

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II. Copula-based Collaborative Structural Damage Diagnosis and Prognosis

A. Conventional Individual-oriented Particle Filter for Fatigue Crack Growth

For a fatigue crack growth process, the evolution of the crack length $\mathbf{a}$ is modeled as:

$$\frac{d\mathbf{a}}{dN} = f(\Delta\mathbf{K}, \mathbf{a}; \boldsymbol{\mu}) \tag{1}$$

where $\frac{d\mathbf{a}}{dN}$ are the increments of crack lengths per cycle, $\Delta\mathbf{K}$ is the ranges of stress intensity factor (SIF), $\boldsymbol{\mu}$ is uncertain material parameters.

The augmented state vector is defined as $\mathbf{x}_k = [\mathbf{a}_k, \boldsymbol{\mu}_k]$, and the state space model is modeled as:

$$\mathbf{x}_k = \begin{bmatrix} \mu_k \\ \mathbf{a}_k \end{bmatrix} = \begin{bmatrix} \mu_{k-1} + \omega_{\mu,k} \\ \mathbf{a}_{k-1} + e^{\omega_{\mathbf{a},k}} f(\Delta\mathbf{K}, \mathbf{a}; \mu)\, \Delta N \end{bmatrix} \tag{2}$$

$$\mathbf{y}_k = \mathbf{a}_k + \eta_k \tag{3}$$

where $\mathbf{y}_k$ is the observation, $\omega_{\mu,k}$ is the parameters evolution noise, $\omega_{\mathbf{a},k}$ is the crack growth noise following Gaussian distribution $N\left(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2\right)$, leading to $E(e^{\omega_{\mathbf{a},k}}) = 1$, η_k is the measurement noise subjected to a zero-mean Gaussian distribution.

To effectively track the evolution process of state vector $\mathbf{x}_k$, the following two tasks need to be accomplished by Bayesian inference:

Forward propagation: predict the state vector $\mathbf{x}_k$ according to the state variables $\mathbf{x}_{k-1}$ at the previous time step and the state transition between the two adjacent time steps:

$$p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}) = \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1})\, p(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1})\, d\mathbf{x}_{k-1} \tag{4}$$

Backward inference: updating the joint probability distribution $p(\mathbf{x}_k \mid \mathbf{y}_{1:k})$ of the state variables $\mathbf{x}_k$ when observation $\mathbf{y}_k$ is available:

$$p(\mathbf{x}_k \mid \mathbf{y}_{1:k}) = \frac{p(\mathbf{y}_k \mid \mathbf{x}_k)\, p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1})}{p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1})} \tag{5}$$

where $p(\mathbf{y}_k \mid \mathbf{x}_k)$ is observation likelihood, $p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1})$ is a normalization constant.

Particle filtering utilizes N_s particles to represent the distribution as:

$$p\left(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k}\right) \approx \sum_{i=1}^{N_s} \widetilde{w}_k^{(i)} \delta\left(\boldsymbol{x}_k - \boldsymbol{x}_k^{(i)}\right) \tag{6}$$

where N_s is the number of particles, δ is the Dirac delta function, $\mathbf{x}_k^{(i)}$ is the i_{th} particle, $\widetilde{w}_k^{(i)}$ is the importance weight of i_{th} sample. More details about the PF can be found in [4].

B. Copula Function to Model the Joint Distribution

A copula [7] is a multivariate distribution with uniform margins on the unit interval. In this study, two kinds of copula function are adopted.

The two-dimensional Frank copula is adopted to approximate the joint distribution of the crack lengths of two individual structures, which is defined as::

$$F_\theta^{Fr}\left(u_1, u_2\right) = -\frac{1}{\theta} \log\left(1 + \frac{\left(\exp\left(-\theta u_1\right) - 1\right)\left(\exp\left(-\theta u_2\right) - 1\right)}{\exp(-\theta) - 1}\right) \tag{7}$$

where u_1 and u_2 are the cumulative distribution function (CDF) of the crack length distribution of two individuals, respectively. θ is the correlation parameter.

The Gaussian copula is employed to retain the correlations between the crack length and parameters prior to the "updating by copula" process and recovered it after the process, which is formulated as:

$$F_R^{\text{Gauss}}(u) = \Phi_R\left(\Phi^{-1}\left(u_1\right), \ldots, \Phi^{-1}\left(u_d\right)\right) \tag{8}$$

where Φ^{-1} is the inverse function of the standard normal distribution, $\mathbf{R}$ is the covariance matrix, d is the number of variables.

C. Estimate the Correlation Parameter in the Copula by the Similarity Metric.

The Maximum Mean Discrepancy (MMD) [8] is utilized to measure the similarity of the predicted crack length distribution d_a and the growth parameter distribution d_μ between two individuals:

$$d_k^2(p, q) = \left\|\mathbf{E}_p\left[\phi\left(\mathbf{x}^1\right)\right] - \mathbf{E}_q\left[\phi\left(\mathbf{x}^2\right)\right]\right\|_{\mathcal{H}_k}^2 \tag{9}$$

where $\mathbf{x}^1$ and $\mathbf{x}^2$ are the two distribution to compare, $\mathcal{H}$ is the endowed reproducing kernel Hilbert space (RKHS) with a kernel ϕ, $\mathbf{E}_{\mathbf{x}\sim p} f(\mathbf{x}) = \langle f(\mathbf{x}), \mu(p)\rangle_{\mathcal{H}}$ is the mean embedding of distribution p in $\mathcal{H}$, $d_k^2(p, q) = 0$ if and only if $p = q$.

Then a simple heuristic approach is adopted to convert the distance metric into a similarity measure as follows:

$$\theta = \theta_0 \times \left(d_0 - \left(\alpha d_a + (1-\alpha) d_\mu\right)\right) \tag{10}$$

where θ_0, d_0 are hyperparameters need to be adjusted. The weighting factor, α, is employed to balance the weight of the d_a and d_μ.

D. Complete Procedure for the Copula-based Joint Diagnosis and Prognosis

The complete procedure of the proposed approach is shown in Algorithm 1.

Algorithm 1 Copula-based Particle filter for multi-structure diagnosis and prognosis

```
Initialization of the fleet with M individuals
Generate particles set x^i_{0,m} {i=1}^{N_s}, m = 1, ..., M from the prior distribution
While during service: do
    For m = 1 : M do
        Predict the damage state for each structure:
        Draw predicted particles x^i_{k,m} ~ p(x_{k,m} | x^i_{k-1,m}), i = 1, ..., N_s
        If structure m observed: then
            Update structure m by the observation z_{k,m}:.
            Evaluate weight of particle w~^i_{k,m} = w^i_{k,m} p(z_{k,m} | x^i_{k,m}), and normalize w^i_{k,m} = w~^i_{k,m} / sum_{q=1}^{N} w~^q_{k,m}, i = 1, ..., N_s
            Generate new particles {x^j_{k,m}}_{j=1}^{N_s} by resampling from x^i_{k,m} {i=1}^{N_s}, and set weight w^j_{k,m} = 1/N_s
            For l = 1 : M and l ≠ m: do
                Update the crack length distribution of structure l by the copula and observation z_{k,m}:
                Fit and store the correlation matrix R of the crack length and parameter distributions by the Gaussian copula function
                Fit the predicted crack length distribution a_{k,l} and a_{k,m} by KDE and calculate the CDF of both distributions.
                Measure the distribution similarity d_a and d_μ by MMD and determine the θ of the copula function.
                Sampling from the copula to obtain {u^i_k}_{j=1}^{N_s} and use the inverse CDF to generate new particles {a^j_{k,(l,m)}}_{j=1}^{N_s} subjected to the defined joint distribution.
                Calculate the weight and resample particles {a^j_{k,(l,m)}}_{j=1}^{N_s} with the observation z_{k,m}
                Extract the resampled crack length {a^j_{k,l}}_{j=1}^{N_s} as the updated crack length distribution.
                Regenerate the particles {x^j_{k,l}}_{j=1}^{N_s} by the stored correlation matrix R of the crack length and parameter distributions.
            End For
        End If
    End For
End While
```

At the beginning, the digital twins of M individual structures within a fleet are initialized with the prior distribution of the crack length and uncertain parameters. During the service, prognoses are conducted simultaneously for each structure based on the monitored loads as input. Upon the inspection of an individual structure in the fleet, the diagnosis step is employed to update the distribution of the uncertain parameters, similar to that of the traditional individual-oriented particle filter. In this study, a copula-based updating step is included, wherein the copula function is utilized to approximate the joint distribution of the crack lengths in two individuals, as detailed in Fig. 1.

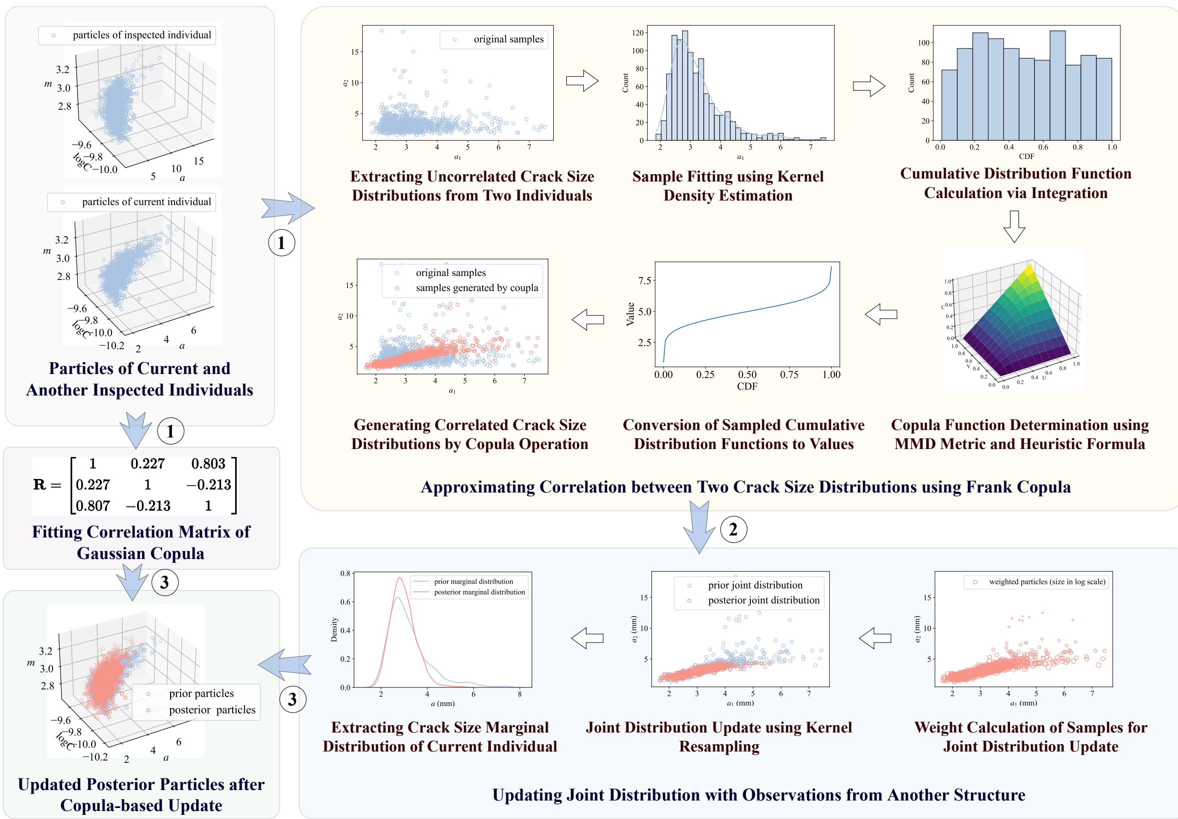


Fig. 1 Flowchart illustrating the generation of new samples incorporating correlation between two individuals via the copula-based method.

III. Validation of the Proposed Framework on Hypothesized and Experimental Datasets

A. Hypothesized Dataset Generation

A simple infinitely large plate with crack growth at the edge of the hole subjected to a bidirectional uniform positive pressure is used to demonstrate the proposed approach. The stress intensity factor range ΔK, is calculated by:

$$\Delta K = \Delta\sigma\sqrt{\pi a} \tag{11}$$

where $\Delta\sigma$ is the stress range.

The Paris law is adopted as the crack growth model.

$$\frac{da}{dN} = C(\Delta K)^m \tag{12}$$

where C and m are material parameters in the Paris law.

The true parameters of the three specimens are presented in Table 1.

Table 1 True parameters of the three hypothesized specimens

Specimen	$a_{0,\text{true}}$	$\log C$	m
1	2.0	-9.82	2.97
2	2.05	-9.85	3.01
3	1.95	-9.79	3.02

B. Fatigue Testing Dataset

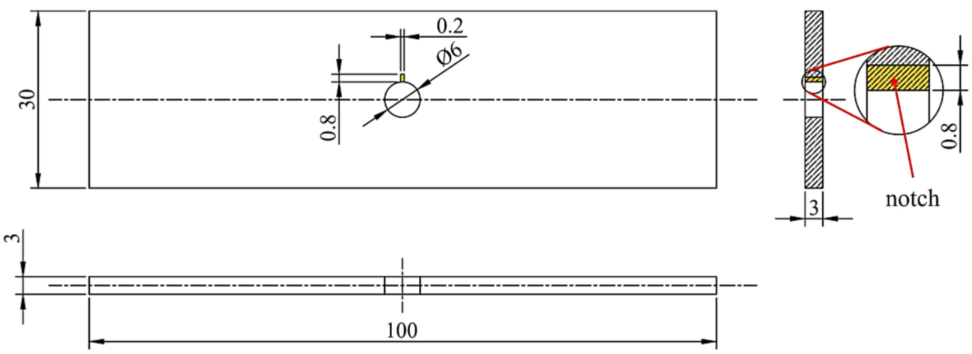
The second dataset is an experimental dataset comprising crack growth histories of 2024 aluminum alloy specimens with center holes [9] as shown in Fig. 2. Three specimens, labeled as #1, #2, and #4, are chosen for analysis. The observations are presented in Fig. 2c, with certain data points intentionally omitted to simulate inspections at varying intervals.

C. Parameters Setting

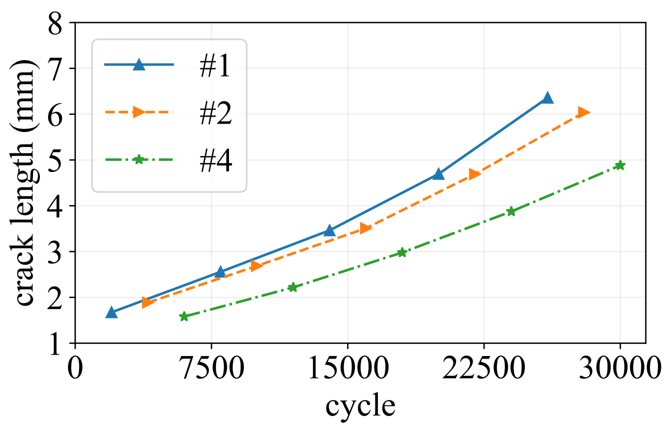
The parameter setting of the proposed approach and the traditional particle filter are shown in Table 2

Table 2 Parameter setting of the particle filter

Parameters	Meaning	Hypothesized	Experimental
a_0	Prior distribution of the initial crack length	$N \sim (2, 0.1^2)$	$N \sim (1.2, 0.1^2)$
$\log C_0$	Prior distribution of the $\log C$	$N \sim (-9.8, 0.1^2)$	$U \sim (-11.8, -10.8)$
m_0	Prior distribution of the m	$U \sim (2.8, 3.2)$	$N \sim (2.65, 0.05^2)$
$\Delta\sigma$	Stress range	25	118.125
η	Measurement noise	$N \sim (0, 0.5^2)$	$N \sim (0, 0.1^2)$
ω_a	noise of the crack growth	$N \sim (-\frac{0.05^2}{2}, 0.05^2)$	$N \sim (-\frac{0.05^2}{2}, 0.05^2)$
ω_1	noise in the evolution of $\log C$	$N \sim (0, 0.01^2)$	$N \sim (0, 0.01^2)$
ω_2	noise in the evolution of m	$N \sim (0, 0.01^2)$	$N \sim (0, 0.01^2)$
N_s	Number of particles	1000	1000
ΔN	Increment of loading cycle	1000	500
θ_0	hyperparameters for the correlation metric	15	2
d_0	hyperparameters for the correlation metric	0.8	1
α	weighting factor for the similarity metric	0.5	0.5



(a) Specimen of 2024 aluminum alloy with prefabricated notch (unit: mm)



(b) Experiment

(c) Selected crack growth results in this study

Fig. 2 Specimen and experiment crack growth results used in this study. (a) and (b) are adapted with permission from [9]. Copyright 2023 Elsevier Ltd.

IV. Results and Discussion

The diagnosis and prognosis results of the two datasets are shown in Fig. 3, in which the proposed approach is compared with the baseline approach that was updated separately. It is observed that the reduction of uncertainty in the absence of observations is attributed to the copula-based updating, resulting in an overall decrease of uncertainties during the full crack growth processes. The improvement in prediction accuracy of the hypothesized dataset is also shown in Table 3, where the prediction error is listed and shown to be lower for the proposed approach compared to the baseline approach.

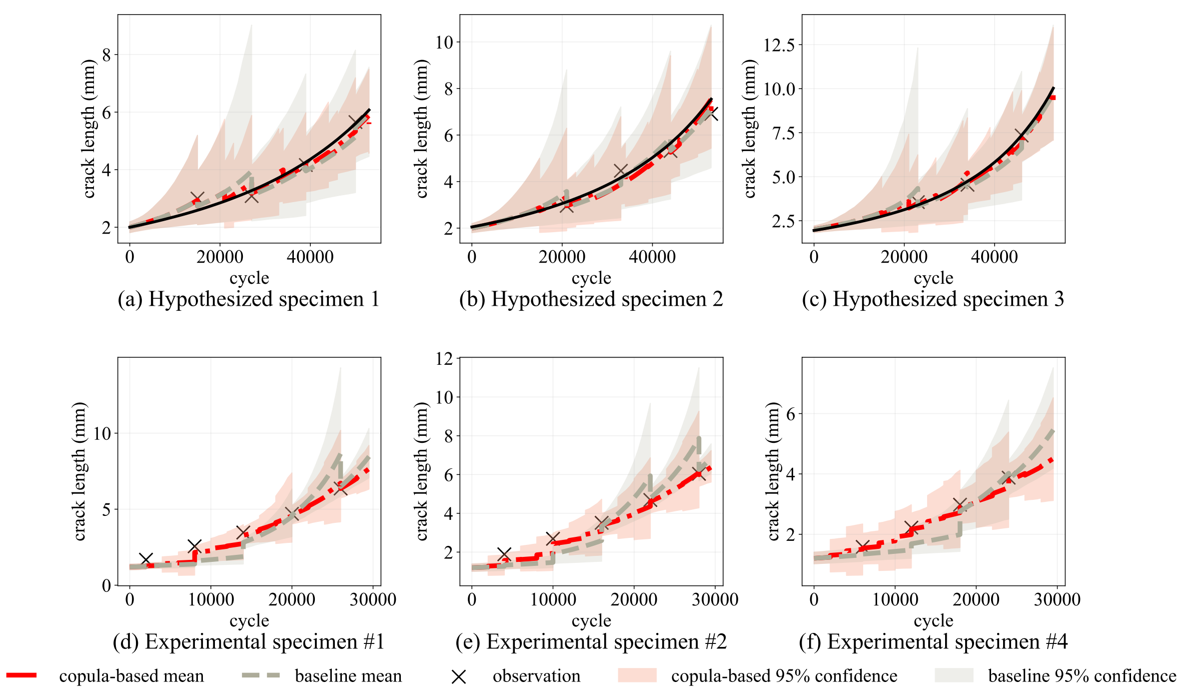
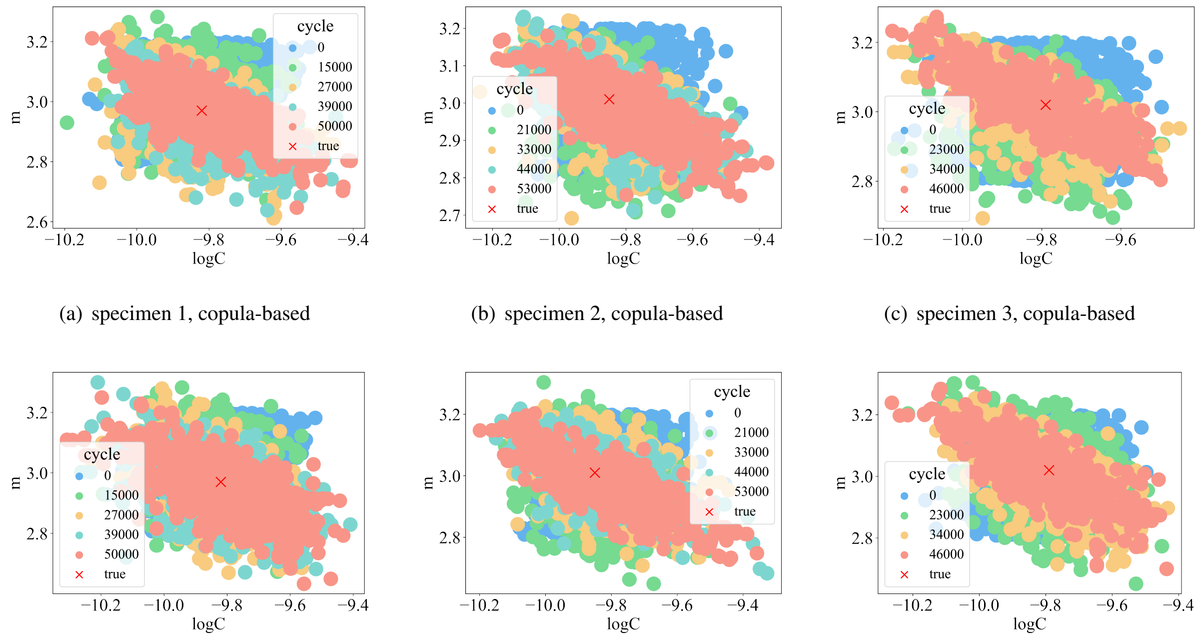


Fig. 3 Comparison of the diagnosis and prognosis results between the proposed copula-based approach and the baseline approach that updated separately.

Table 3 Comparison of the prediction error (RMSE)

Specimen	Copula-based	baseline
1	**1.149**	1.462
2	**1.157**	1.309
3	**1.176**	1.616

The update of material parameters of the hypothesized dataset is presented in Fig. 4. The results of the proposed method are in close agreement with the baseline method. It should be noted that the Copula function in this study is only used to establish the relationship of damage states between different individuals without considering the crack growth parameters. However, this also indicates that the Gaussian Copula function effectively captures the correlation between individual crack length and the distribution of material parameters in crack growth. As a result, there is a significant reduction in uncertainty in the crack growth parameters when updated by inspection results of the individual itself.



(a) specimen 1, copula-based

(b) specimen 2, copula-based

(c) specimen 3, copula-based

(d) specimen 1, baseline

(e) specimen 2, baseline

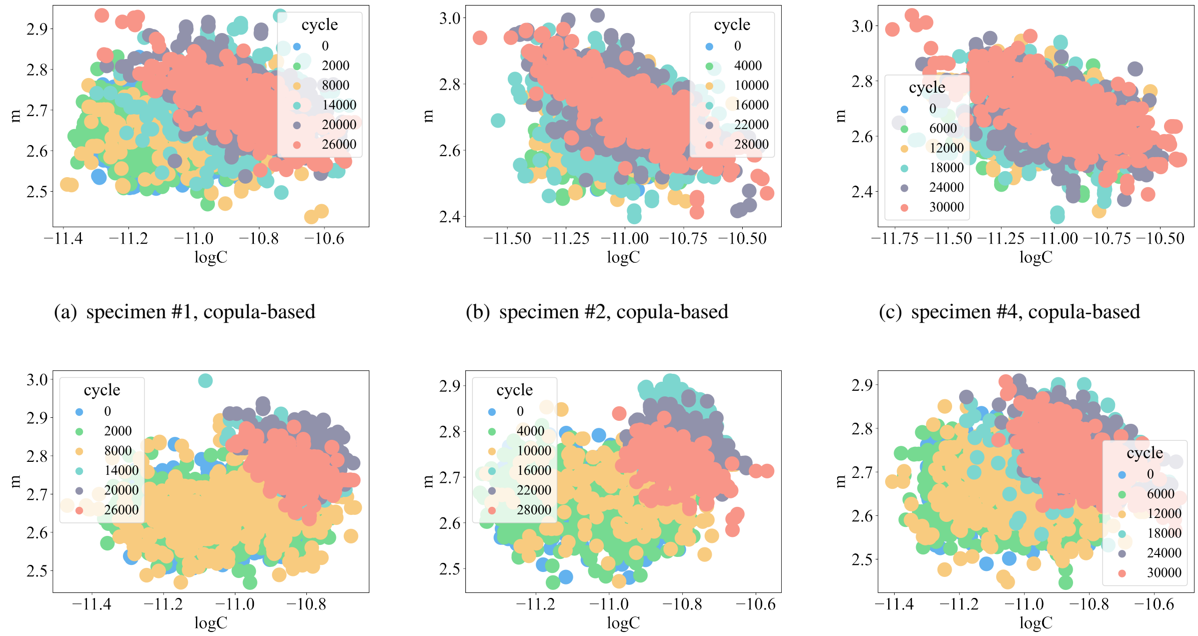
(f) specimen 3, baseline

Fig. 4 Updating of the material parameters logC **and** m

Fig. 5 shows the updating of the material parameters of the experimental dataset. It can be seen that after several updates, the material parameter uncertainty of the benchmark method is smaller than that of the proposed method. Nevertheless, upon closer examination of the second half of the crack growth process in Fig. 3, it becomes evident that the crack growth of the proposed method is more realistic, indicating that the mean values of the crack growth parameters may be more accurate.

V. Conclusion

In this paper, a novel copula-based approach is proposed to provide a promising solution for the collaborative diagnosis and prognosis of multiple structures within a fleet. The approach can effectively update the damage state of uninspected structures by utilizing the inspection results of other structures. The results of the two case studies demonstrate the effectiveness of coupling the damage states of different structures within a fleet using copula functions, both in hypothetical and real experimental datasets. These findings are significant in the context of developing a fleet maintenance digital twin, given the high cost of inspecting and maintaining aircraft structures.



(a) specimen #1, copula-based

(b) specimen #2, copula-based

(c) specimen #4, copula-based

(d) specimen #1, baseline

(e) specimen #2, baseline

(f) specimen #4, baseline

Fig. 5 Updating of the material parameters logC **and** m

This work represents a preliminary exploration of the digital twin for fleet maintenance. Some challenges must be addressed before the method can be applied in the fleet, such as developing more robust methods for measuring the similarity of damage states between complex structures and devising improved inspection interval strategies that align with the method and thus enhance overall diagnostic and predictive efficiency, among others.

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